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ABSTRACT

The purpose of this study was to design a unit on percent for adult remedial students based on the principles of Piaget's theory and to evaluate the effectiveness of the unit as a means of improving one characteristic of formal operations, the ability to use porportion. The subjects consisted of 89 undergraduate remedial mathematics students at a branch of the City University of New York. Results indicate that there seems to be a relationship between learning percent and improving the ability to use proportions. Moreover, it would seem that an instructional unit can be developed which implements some of the heuristic principles of Piaget's theory and which simultaneously improves one characteristic of formal operations and develops basic mathematical principles. In addition, it appears that Piaget's theory can be successfully applied to the instruction of adult remedial students and that this theory can be implemented in an actual classroom situation involving instructors of remedial mathematics courses. (Author/MK)

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IMPROVING ONE CHARACTERISTIC OF FORMAL OPERATIONS IN ADULT REMEDIAL STUDENTS THROUGH A PIAGETIAN ORIENTED INSTRUCTIONAL UNIT ON PERCENT

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Piaget's work represents a comprehensive study of the evolution of thinking in children and adolescents. His theory of intellectual development describes the progressive construction of operational structures from birth to maturity, where the highest levels of logical thinking are characteristic of the formal operations period. In order for an individual to have reached this stage, he must have passed through the previous phases of logical thinking in his development. Thus, Piaget's theory of intelligence may be considered a model for the development of the intelligent human adult. However, certain studies (Abercombie, 1960; McKinnen and Renner, 1971, 1974; Griffiths, 1974; Griffiths, 1973) have suggested that many adults do not reach the formal operations level. Moreover, research has offered relatively little in terms of how to nurture and enhance a person's ability to use formal logical thinking skills. Piaget himself does not provide a systematic application of his theory to education and despite the huge amount of Piagiatian-related research and the outstanding progress made in developing a framework of genetic epistomology based on his theory, a great deal more work is needed in terms of classroom applications of Piaget's theory at the adolescent and adult levels (Van Engen, 1971).

Much of Piaget's research has involved the study of the development of mathematical concepts. Since his theory seeks to explain how mental operations basic to mathematical thought develop, mathematicians and mathematics educators are especially interested in his work. Many studies concerning the mathematical development of young children have been conducted, but relatively little work has been done for secondary school youths or adults (Dessart and Frandsen, 1974), even though the logical

skills of the formal operations period can be correlated with the basic mathematical competencies associated with secondary school and remedial mathematics curricula. For example, the ability to operate with operations and to form relations with relations are involved in mathematical proportionality and other related problems. The mastery of percent involves the ability to use and understand proportions, a logical thinking skill. Moreover, Piaget's group of four operations or the INRC group (identity, negation, reciprocal, correlate) occurs in all problems of proportionality. However, the results of the 1973 National Assessment of Educational Progress (NAEP) in Mathematics found that students performed badly in exercises about or using percent. The most recent finding of the National Assessment of Education Progress (NAEP) indicated a continued difficiency in problem solving and other skills applications. Thus, there appears to be a clearly defined need to improve logical thinking skills on the part of adolescents and young adults. Moreover, the rise in the need for remedial mathematics courses in colleges across the country indicates further a need to improve mathematical competencies and logical thinking skills on the part of adults.

Data gathered from diverse Piagetian training studies have shown that an improvement in performance may be expected where training is provided for those who would be near a particular stage of development (Beilin, 1971). Since adults should be at or near the formal operations level, training should lead to an improvement of a formal operations characteristic. Thus, given the correlation between the logical skills of the formal operations period and mathematical competencies, especially in terms of the relationship of percent to proportions, it would seem

possible to incorporate into an instructional unit the development of the ability to work with percent and the improvement of a logical thinking skill.

Although the attention directed towards Piaget's theory has been in terms of the development of cognitive processes in children, the theory, nevertheless, is an explanation of the predecessors of adult knowledge. Within this context, all individuals must pass through the same sequence of stages of development in order to achieve the characteristics identifying the final period of development, formal operations, or adult levels of logical thinking. Since the theory is explicitly developmental, its principles would seem applicable to an instructional unit for adults geared toward the improvement of a logical thinking skill as well as the mastery of certain mathematical concepts.

Statement of the Problem

The purpose of this study was to design a unit on percent for adult remedial students based on the principles of Piaget's theory and to evaluate the effectiveness of the Unit as a means of improving one characteristic of formal operations, the ability to use proportion.

More specifically, the investigator sought to answer the following questions:

1. Did significant differences exist between the experimental and control groups in the mastery of the cognitive objectives generally prescribed for a unit on percent for adult remedial mathematics students as measured by the scores on the Unit tests?
2. Did significant differences exist between the experimental and control groups in the ability to use proportions as measured by the scores on pretest and posttest on proportions?

SUBJECTS, MATERIALS AND PROCEDURES

Subjects

The subjects for this study consisted of 89 undergraduate remedial mathematics students at a branch of the City University of New York. The majority of students were freshmen, 20-24 years of age, enrolled in the first undergraduate remedial mathematics course offered by the mathematics department of the college. The course was a one credit course, meeting five hours per week for approximately fifteen weeks, designed for entering college students who had been determined by the college to be in need of mathematics remediation. Criteria for this determination was:

1. they had not completed elementary algebra in high school; or
2. they had been out of high school for a considerable length of time and voluntarily elected to begin with that course; or
3. they had completed elementary algebra in high school but went back to this course after doing poorly in a more advanced course.

Six sections of the course with a total registration of 123 students constituted the sample for the study. Three sections served as the experimental group. The sections of this group were determined by the instructors' willingness to participate in the experiment. The control group consisted of three other randomly selected sections of the course. Of the initial 123 students in the sample,89 were actually used by the investigator as subjects for the study. Only students who had taken both the pretests and posttests and who were not absent for more than one class session on percent were included in the final group. Thus, there were 48 students in the final experimental group and 41 students in the final control group.

Materials

The materials used in this study were: the Unit on Percent, the Unit test and pretest on percent and the pretest and posttest on the ability to use proportions.

Unit on Percent

The Unit on Percent developed by the investigator consisted of five sections: a general review of Piaget's theory in relation to formal operations; proportionality and the INRC group; explication of Piaget's theory in relation to the specific techniques developed for teaching percent; a Teacher's Guide; and student worksheets. The Unit on Percent was specifically designed not only to provide teachers with an instructional guide but also to provide an explanation of the theoretical principles involved in the Unit's development of the concept of percent. The student worksheets, devised by the investigator, were intended to help students in the experimental group review the ideas presented in each section of the instructional unit.

In order to develop the Unit, the investigator conducted an extensive review of the literature concerning Piaget's theory and its applications to teaching as well as a review of available remedial mathematics curriculum. Based on this review, the techniques employed in teaching the concept of percent involved: a sequential development beginning with concrete ideas defined through examples; the use of initial problems which required students to raise questions or predict outcomes and interact with one another and the teacher; an explanation, presented by the teacher, of the reasoning employed in examining evidence presented by problems and in considering alternative strategies for their solution.

A distinguishing characteristic of the instructional unit was the initial use of one-to-one correspondence, in terms of the part to whole relationships, to solve specific percent problems and introduce the use of proportions. In addition, since research (Abramowitz, 1975) has suggested that success on proportionality tasks are correlated to an understanding of fractions, the Unit on Percent included a detailed analysis of equal fractions, thus enabling students to better understand and use proportions in solving percent problems.

The Unit was subjected to initial trial and evaluation by means of a pilot study. In addition, after the pilot study, the Unit was sent to a Jury of Experts, concerning the content and validation of the Unit. Given the nature of the Unit, the individual's evaluating the materials had to be knowledgable in Piaget's theory, mathematics, remedial mathematics for adults, and mathematics education. Since a single individual cannot be thoroughly informed in all these areas, the Jury of Experts was chosen by the investigator so that each individual had expertise in at least two of the areas cited. In order to facilitate the task of the Jury of Experts, a questionnaire was prepared by the investigator. All of the responses received validated the Unit.

Pretests and Posttests

The Unit test was a comprehensive test on percent designed by the investigator to measure the cognitive objectives generally prescribed for a Unit on Percent for adult remedial students. An analagous form of the Unit test constituted the pretest on percent. The pretest and posttest on the ability to use proportions was a revised version of the tests developed by Lovell and Butterworth (1966) concerning the ability

to handle metric proportions. Written permission from Dr. Lovell for this revision and his evaluation of the revised pretest and posttest were obtained. These tests were also validated by the Jury of Experts.

The reliability of the testing instruments were found using the split-half method. ($r(75) = .94$, $p < .01$ for the tests on percent and $r(75) = .97$, $p < .01$ for the tests on proportion.)

Procedures

The procedures utilized in this study involved a pilot study, a field experiment and a statistical analysis.

Pilot Study

In order to subject the Unit on Percent and the testing instruments to initial trial and evaluation, a pilot study was conducted by the investigator at a branch of the City University of New York. The subjects used for the pilot study were 20 undergraduate students in an elementary mathematics education class. These students had been tested in basic mathematical skills by the instructor of the class and had been found to be in need of mathematics remediation especially in the area of percent.

The results of the pilot study supported the effectiveness of the Unit on Percent in terms of the mastery of percent and the improvement of the ability to use proportions. This initial trial also suggested some revisions in the Teacher's Guide and student worksheets which were incorporated into the final Unit on Percent.

Field Experiment

The final validated Unit on Percent was tested in three sections of the first remedial mathematics course offered at a branch of the City University of New York. All instructors of the course were informed by

the investigator of the nature and purpose of the experiment. Three instructors were willing to implement the Unit on Percent developed by the investigator in their classrooms. The classes of these instructors constituted the experimental group for this investigation. After the experimental classes were chosen, three other sections of the course were randomly selected from those sections of the course whose instructors did not wish to teach the Unit on Percent developed by the investigator. These three classes constituted the control group for this study.

All the classes in the experimental group met in the morning. Two classes in the control group met in the morning and one in the afternoon. The instructors of the experimental classes consisted of one female and two male teachers, 23 to 27 years of age. Two of the instructors had taught the course more than once and each had at least one year of teaching experience on the college level. The instructors of the control group were all males, 25 to 38 years of age. Two of the instructors had taught the course more than once and each had at least two years of teaching experience on the college level.

The instructors of the experimental classes attended teacher training sessions conducted by the investigator once a week for three weeks prior to the beginning of classes. Prior to the first training session, these teachers were provided with the Unit on Percent. The teacher-training sessions were intended to acquaint the instructors with the theoretical framework of the Unit and to explain and demonstrate the implementation of the teacher's guide. The instructors also demonstrated the lessons in the Unit by means of classroom simulations so as to maintain as little variation as possible in the actual instruction of the Unit by the teachers.

At the last training session each experimental instructor was given sufficient copies of the pretest and posttest on percent, pretest and posttest on the ability to use proportions, and each of the four student worksheets. The procedures for administering these tests were explained and any final questions were answered. At the same time, each instructor of the control classes received sufficient copies of the pretest and posttest on percent and the pretest and posttest on the ability to use proportions along with a written explanation of the administration of these tests.

The Unit on Percent developed by the investigator was implemented in the experimental classes during the same week that the control classes were beginning the Unit on Percent as presented in the course curriculum. The instructors of both the experimental and control classes devoted five class lectures to instruction in the Unit on Percent with their classes. In the experimental classes, student worksheets were distributed as homework assignments to the students at the end of each lecture, which completed the lessons in the teacher's guide. Part of each subsequent lecture was devoted to answering student questions concerning problems on the worksheet.

The investigator observed two class sessions of each experimental and control class during the implementation of the Unit on Percent. Given the differences in the scheduling of classes, both groups appeared to proceed at the same pace. The observed experimental class sessions were adhering to the teacher's guide developed by the investigator. The Unit on Percent for both the experimental and control groups was completed within the same week.

The pretest on percent and the ability to use proportions was administered to both the experimental and control groups at the class session immediately preceeding the instruction of the Unit on Percent. Each test took 25 minutes to administer. The experimental and control groups took the posttest on percent and the ability to use proportions during the class session immediately following completion of the instruction of the Unit on Percent. Each test took 25 minutes to administer. The investigator hand scored the pretests and posttests of the experimental and control groups according to the number of items answered correctly on each test.

Statistical Procedures

The statistical procedures utilized in this investigation consisted of three parts 1) comparison of pretest scores for experimental and control group 2) comparison of posttest scores for the experimental and control groups and 3) comparison of pretest and posttests scores for the experimental and control groups, respectively.

The pretest scores on percent and the ability to use proportions were compared by means of the independent t-test. This comparison of pretest scores was made to assure that the experimental and control samples represented the same population for each measure.

In order to determine if significant differences existed between the experimental and control groups in the ability to use proportions and in the mastery of percent, the posttest scores on percent and the ability to use proportions were compared by means of the independent t-test.

The pretest and posttest scores on percent and the ability to use proportions in the experimental and control groups respectively were compared by a correlated data t-test in order to determine in which direction

the change in scores occured in each group. The correlated data t-test is the analagous form of the analysis of covariance for two groups which is the statistical method suggested by Campbell and Stanley (1968, p.49) for a modified Solomon's design, the research design used for this study. Dr. Fred Streit, visiting professor for research at Rutgers University, was consulted concerning the most appropriate method of statistical analysis for this study. Upon his advise the statistical procedures as previously stated were adopted by the investigator. The actual data analysis was also performed by Dr. Streit.

Results and Discussion

Results

Comparison of pretest scores on percent and the ability to use proportions using the independent t-test showed the group means for the experimental and control groups not to be significant, $t(89) = .013$; $t(89) = .077$, $p < .01$. The experimental and control groups were therefore determined to be comparable on these two measures.

Comparison of posttest scores on percent using the independent t-test showed significant differences between the experimental and control groups at the .001 level of confidence ($t(89) = 4.08$, $p < .001$). These determined differences favored the experimental group. It was therefore determined that the experimental group who used the Unit on Percent developed by the investigator scored higher on the posttest on percent than did the control group. The same comparison of posttest scores on the ability to use proportions showed significant differences between the experimental and control groups at the .004 level of confidence ($t(89) = 2.93$, $p < .004$). These differences again favored the experimental group. It was therefore concluded

that the experimental group scored higher on the posttest on the ability to use proportions than did the control group.

Since the comparison of posttest scores on percent and the ability to use proportions between the experimental and control groups indicated significant differences, the comparison of pretest and posttest scores on the two measure, for each group, experimental and control, was made to determine the direction in which the change had occurred in each group. The pretest and posttest scores for the experimental group on percent and the ability to use proportions, respectively, were compared by means of the t-test for correlated data. The investigator determined that there were significant differences between pretest and posttest scores ($t(89)$= 12.156, $p < .001$; $t(89) = 7.887$, $p < .001$). These differences favored the posttest scores. The investigator therefore concluded that the experimental group scored significantly higher on the posttest on percent and the ability to use proportions than this group had scored on the pretests of these measures. Using the same analysis for the control group, significant differences were found between the pretest and posttest scores ($t(89)=9.004$, $p < .001$; $t(89)=4.421$, $p < .001$). Again, the differences favored the post-test scores. The investigator therefore concluded that the control group scored significantly higher on the posttest on percent and the ability to use proportions than this group had scored on the pretests of these measures. The comparison of posttest scores between the experimental and control groups, however, indicated that the experimental group showed a significantly greater gain from pretest to posttest than did the control group.

Conclusions

On the basis of the findings, the following conclusions were made by the investigator:

1. The experimental group using the Unit on Percent scored significantly higher than the control group on the posttest on percent.
2. The experimental group using the Unit on Percent scored significantly higher than the control group on the posttest on the ability to use proportions.

Discussion

The study has all the limitations attendant upon volunteer instructors and the lack of absolute control of all extraneous variables. Nonetheless, given the limitations of the study there would seem to be a relationship between learning percent and improving the ability to use proportions. Moreover, it would seem that an instructional unit can be developed which implements some of the heuristic principles of Piaget's theory and which simultaneously improves one characteristic of formal operations and develops basic mathematical principles. In addition, it appears that Piaget's theory can be successfully applied to the mathematics instruction of adult remedial students and that this theory can be implemented in an actual classroom situation involving instructors of remedial mathematics courses.

Significance is indicated as the study addresses several areas of concern. A review of the literature shows little evidence that any theoretical framework or learning theory has been considered as a basis for developing alternative teaching techniques for adult remedial students even though remedial mathematics has become a necessary component of high school and college curricula. Research also offers very little in terms of ways to nurture and enhance a person's ability to use formal logical thinking skills, while the most recent results of the National Assessment of Educational Progress in Mathematics clearly indicates the need to improve such skills.

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